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Department of Mathematical Sciences

Titles and abstracts of talks

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Almost matching bounds for tail probabilities for integrals along fractional Brownian motion

Abstract: Consider a differential equation driven by fractional Brownian motion with smooth bounded vector fields. Cass-Hairer-Litterer-Tindel contained implicitly an upper bound for tail probability (made explicit by Baudoin-Nualart-Ouyang-Tindel). We are interested in the sharpness of this upper bound. In particular, we studied an equation where the solution can be explicitly expressed as an integral and showed that the tail probability matches with the known upper bound up to an arbitrarily small power in the exponential. Joint work with Xi Geng.

Zdzisław BRZEŹNIAK

TBA

Thomas CASS

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(Expected) signatures, weighted signature kernels and functions on unparameterised paths

Abstract: The signature is a non-commutative exponential that appeared in the foundational work of K-T Chen in the 1950s. It is also a fundamental object in the theory of rough paths (Lyons, 1998). More recently, it has been proposed and used as part of a practical methodology to give a way of summarising multimodal, possibly irregularly-sampled, time-ordered data in a way that is insensitive to its parameterisation. A key property underpinning this approach is the ability of linear functionals of the signature to approximate arbitrarily any compactly supported and continuous function on (unparameterised) path space. We present some new results on the properties of a selection of topologies on the space of unparameterised paths. Relatedly, we review some recent innovations in the theory of the signature kernel by introducing and analysing the properties of a family of so-called weighted signature kernels

The talk will draw on material from two recent papers; one is joint work with William F. Turner, the other is a joint work with Terry Lyons and Xingcheng Xu.

Mickaël CHEKROUN

TBA

Hugo CHU

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The conditioned Lyapunov spectrum for random dynamical systems

Abstract: We establish the existence of a full spectrum of Lyapunov exponents for memoryless random dynamical systems with absorption. To this end, we crucially embed the process conditioned to never being absorbed, the Q-process, into the framework of random dynamical systems, allowing us to study multiplicative ergodic properties. We show that the finite-time Lyapunov exponents converge in conditional probability and apply our results to iterated function systems and stochastic differential equations.

This is joint work with Matheus De Castro, Jeroen Lamb, Martin Rasmussen (Imperial College London), Dennis Chemnitz and Maximilian Engel (Freie Universität Berlin).

Giuseppe DA PRATO

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An application of macroscopic fluctuation theory to reaction-diffusion equations

Abstract: We consider the Gibbs measure μ_ϵ in the Hilbert space $H = L^2(0,1)$ associated with a reaction–diffusion equation $dX = (\Delta X + p(X))dt + \sqrt{\epsilon}\, dW(t)$, Δ being the Laplace operator on $[0,1]$ equipped with Dirichlet boundary conditions, p a polynomial of odd degree having negative leading coefficient and W an H–valued cylindrical process. As well known, this Gibbs measure describes a reversible system. We perturb this system by a term $b(X)dt$ that is not of a potential form obtaining a system which is no more reversible and which possesses an invariant measure ν_ϵ. We give a characterisation of the generator of the adjoint transition semigroup and we show that in general it is not associated with a stochastic PDE (the adjoint dynamics). We denote by $\overline{X_\epsilon}$, a stationary process and by $\overline{X^*_\epsilon}$ its reversed process. Inspired by the recent paper *Macroscopic Fluctuation Theory* from L. Bertini, A. De Sole, D. Gabrielli, G. Jona Lasinio and C. Landim, we describe both the relaxation of a macroscopic fluctuation to equilibrium and the spontaneous emergence from the equilibrium. These results are achieved using large deviations, for the process $\overline{X_\epsilon}$ in the first case and for $\overline{X^*_\epsilon}$ in the second.

Work in progress with Stefano Bonaccorsi, Sandra Cerrai and Luciano Tubaro.

Zhao DONG

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Large time behavior of solution for stochastic Burgers equation

Abstract: We consider the large time behavior of strong solutions to a kind of stochastic Burgers equationt, that is, the rarefaction wave is still stable under white noise perturbation and the viscous shock is not stable yet. Moreover, a time-convergence rate toward the rarefaction wave is obtained.

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The stochastic gradient descent algorithm for machine learning

Abstract: The stochastic gradient descent algorithm (SGD) is the most popular training algorithm in machine learning. In this lecture, we discuss some basic theoretical issues of this algorithm, including

- the convergence behavior
- the optimal learning rate and batch size
- in the over-parametrized regime, the selection of the particular global minimum that SGD converges to

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Convergence to stationary solutions in singular quasilinear stochastic PDEs

Abstract: A certain quasilinear PDE is derived from zero-range process (interacting random walks) in a regularized Sinai-type random environment via hydrodynamic limit; see Landim, Pacheco, Sethuraman and Xue (AAP, to appear). Then, in a general setting including this PDE, F, Hoshino, Sethuraman and Xie (AIHP, 2021) show that the PDE leads to a singular quasilinear stochastic PDE defined in a paracontrolled sense by removing the regularization.

For the singular quasilinear stochastic PDE obtained in this way, we discuss the global-in-time solvability and the convergence as time tends to infinity to stationary solutions, which are different for each conserved quantity. The equation has a striking character, the initial layer property of improving regularity, under a certain nonlinear transform. This is joint work with Bin Xie and appeared in Stoch. PDE: Anal. Comp., online, 2022.

István GYÖNGY
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Filtering of partially observed jump diffusions

Abstract: Partially observed jump diffusions, described by stochastic differential equations driven by Wiener processes and Poisson random measures are considered. In the first part of the talk the filtering equations, i.e., the equations for the conditional distribution $P_t(dx)$ and for the unnormalised conditional distribution of the unobserved component at time t, given the observations until t, are presented. These are (possibly) degenerate stochastic integro-differential equations. By the help of these equations new results on the existence and on the regularity properties of the conditional density $\pi_t = P_t(dx)/dx$ are established in the second part of the talk. The talk is based on a joint work with Fabian Germ and Alexander Davie in Edinburgh University.

Martin HAIRER
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A mathematical journey through scales

Abstract: The tiny world of particles and atoms and the gigantic world of the entire universe are separated by about forty orders of magnitude. As we move from one to the other, the laws of nature can behave in drastically different ways, sometimes obeying quantum physics, general relativity, or Newton's classical mechanics, not to mention other intermediate theories. Understanding the transformations that take place from one scale to another is one of the great classical questions in mathematics and theoretical physics, one that still hasn't been fully resolved. In this lecture, we will explore how these questions still inform and motivate interesting problems in probability theory and why so-called toy models, despite their superficially playful character, can sometimes lead to certain quantitative predictions.

N.V. KRYLOV
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On strong solutions of time inhomogeneous Itô's equations with Morrey diffusion and drift. A supercritical case

Abstract: We prove strong existence and uniqueness of solutions of Itô's stochastic time dependent equations with irregular diffusion and drift terms of Morrey class type.

Chris KUEHN
TBA

Oana LANG
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Analytical and numerical properties for a stochastic transport 2D Euler equation

Abstract: In this talk I will present a methodology for proving existence of a unique global strong solution for a stochastic two-dimensional Euler vorticity equation driven by noise of transport type. In particular, I will show that the initial smoothness of the solution is preserved, using an approach based on a liniarised approximating sequence. In the second part of the talk I will introduce a probabilistic pathwise approach to effectively calibrate the stochastic vorticity equation mentioned above. I will show that the driving stochastic transport parameters can be calibrated in an optimal way to match a set of given data, and the model is robust with respect to these parameters.

This work is based on:

[1] *Well-posedness for a stochastic 2D Euler equation with transport noise*, Stochastics and Partial Differential Equations: Analysis and Computations, 1-48 (2022), joint with Dan Crisan.

[2] *A pathwise parameterisation for stochastic transport*, to appear in the STUOD Springer Proceedings, joint with Wei Pan.

Xue-Mei LI
TBA

Zeng LIAN

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Random Horseshoe of Anosov systems driven by a quasi-periodic forcing

Abstract: Consider C^2 Anosov systems on a compact manifold driven by a quasi-periodic forcing. We study their dynamical complexity on various levels from both perspectives of path-wise dynamics and stochastic processes. In this talk, I will report the results on the existence of random horseshoe from two different viewpoints: topology and probability. This is joint with Wen Huang and Kening Lu.

Yu LIU

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Numerical approximation to random periodicity of stochastic differential equations

Abstract: In this talk, I will introduce numerical approximation to periodic measure of a time periodic stochastic differential equations under weakly dissipative condition. For this we first study the existence of periodic measure ρ_t and the large time behaviour of $\mathcal{U}(t,s,x) := \mathbb{E}\phi(X_t^{s,x}) - \int \phi d\rho_t$, where $X_t^{s,x}$ is the solution of the SDEs and ϕ is a test function being smooth and of polynomial growth at infinity. With the existence and geometric ergodicity of the periodic measure of the discretized semi-flow from numerical approximation, we obtain weak convergence of order one for Euler-Marumaya scheme. Some numerical experiments will be presented to illustrate our theoretical results. This talk is based on a joint work with Chunrong Feng and Huaizhong Zhao.

Kening LU

TBA

Michela OTTOBRE

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Non-mean-field interacting particle systems and uniform in time averaging

Abstract: We start by presenting results on Poisson equations on non-compact state spaces, with coefficients that can grow super-linearly; we then use such results to obtain a uniform in time averaging theorem for Stochastic Differential Equations (SDEs) with non-Lipschitz coefficients. Key to obtaining both our UiT averaging result and to enable dealing with the super-linear growth of the coefficients (both of the slow-fast system and of the associated Poisson equation) is conquering exponential decay in time of the space-derivatives of appropriate Markov semigroups. We refer to semigroups which enjoy this property as being Strongly Exponentially Stable. As an application we consider a population of N particles interacting with each other through a dynamical network. In turn, the evolution of the network is coupled to the particles' positions. In contrast with the mean-field regime, in which each particle interacts with every other particle, i.e. with $\mathcal{O}(N)$ particles, we consider the a priori more involved case of a sparse network; that is, each particle interacts, on average, with $\mathcal{O}(1)$ particles. We also assume that the network's dynamics is much faster than the particles' dynamics, with the time-scale of the network described by a parameter $\varepsilon > 0$. We combine the averaging ($\varepsilon \to 0$) and the many particles ($N \to \infty$) limits and show that the evolution of the particles' empirical density is described (after taking both limits) by a non-linear Fokker-Planck equation; we moreover give conditions under which such limits can be taken uniformly in time, hence providing a criterion under which the limiting non-linear Fokker-Planck equation is a good approximation of the original system uniformly in time. This is based on joint works with D. Crisan and Ewelina Zatorska (Imperial College), Paul Dobson, Ben Goddard and Iain Souttar (Maxwell Institute), Julien Barre (Orleans).

Shige PENG
TBA

Szymon PESZAT
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On Linear Stochastic Flows

Abstract: The talk is based on a joint work with Ben Goldys, University of Sydney. We study the existence of the stochastic flow associated to a linear stochastic evolution equation

$$dX = AX dt + \sum_k B_k X dW_k,$$

on a Hilbert space. Our first result covers the case where A is the generator of a C_0-semigroup, and (B_k) is a sequence of bounded linear operators such that $\sum_k \|B_k\| < +\infty$. We also provide sufficient conditions for the existence of stochastic flows in the Schatten classes beyond the space of Hilbert-Schmidt operators. Some new results and examples concerning the so-called commutative case are presented as well.

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Parabolic transport type equations arising from turbulence

Abstract: The main problem in the statistical theory of turbulence is to find a good description of distributions of turbulence dynamical variables, such as velocity, vorticity, pressure and so on. These fluid dynamics variables, according to Kolmogorov are random fields, with distributions unlikely being Gaussian. It is therefore a rather challenging scientific program to develop a theory of random fields arising from turbulence. In this talk I report some results about the singletime single-point probability density function (PDF) of the velocity and vorticity fields of a turbulent flow. The PDF PDE is a highly non-linear parabolic-transport equation, which depends on two conditional statistical numerics of important physical significance. The PDF PDE is a general form of the classical Reynolds mean flow equation, and is a precise formulation of the PDF transport equation. The PDF PDE provides us with a new method for modelling turbulence.

Baoyou QU
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Entrance measures of SDEs and the ergodicity through lifting

Abstract: We study the long time behaviour of the transition probabilities of time-inhomogeneous Markov processes. We extend Harris's "small set" method to the time-inhomogeneous situation with the help of Hairer-Mattingly's refinement of Harris's recurrence to a one-step contraction. For stochastic differential equations (SDEs for short) with locally Lipschitz, polynomial growth and one-sided linear growth coefficients, in order to establish the local Doeblin condition, we obtain a nontrivial lower bound estimates for the fundamental solution of the corresponding Fokker-Planck equation. The drift term is allowed to be possibly non-weakly-dissipative, but appear to be weakly dissipative over a large time average. As an application we obtain the existence and uniqueness of quasi-periodic measure. We then lift the Markovian semigroup to a cylinder on a torus and obtain unique invariant measure and its ergodicity. This talk is based on a joint work with Chunrong Feng and Huaizhong Zhao.

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The Law of Large Numbers under Sublinear Expectations

Abstract: We first introduce Stein's method under sublinear expectations, by which we give the convergence rate for the (weak) law of large numbers under sublinear expectations ($\mathbf{LLN}^*$). Then we give a version of strong $\mathbf{LLN}^*$ for a sequence of i.i.d random variables under a sublinear expectation $\mathbf{E} = \sup_{P\in\Theta} E_P$ defined on a Polish space Ω with Θ being weakly compact.

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Computing ergodic limits of reflected diffusions

Abstract: A simple-to-implement weak-sense numerical method to approximate reflected stochastic differential equations (RSDEs) is proposed and analysed. Together with the Monte Carlo technique, it can be used to numerically solve linear parabolic and elliptic PDEs with Robin boundary condition. One of the key results presented is the use of the proposed method for computing ergodic limits, i.e. expectations with respect to the invariant law of RSDEs, both inside a domain and on its boundary. This allows to efficiently sample from distributions with compact support. Both time-averaging and ensemble-averaging estimators are considered and analysed. It is proved that the method has the first order of weak convergence, both at finite time and for approximating ergodic limits. A number of extensions are considered including a second-order weak approximation, the case of arbitrary oblique direction of reflection, and a new adaptive weak scheme to solve a Poisson PDE with Neumann boundary condition. The presented theoretical results are supported by several numerical experiments. Preliminary results on approximation of confined Langevin dynamics will be also discussed. The talk is based on joint works with Ben Leimkuhler (Edinburgh) and Akash Sharma (Nottingham).

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A non-standard approach to non-linear expectation

Key words: non-linear expectation, non-standard analysis, imprecise probability, internal set theory, risk analysis

Abstract: Lower previsions use sets of probability measures, specified through non-linear bounds on expectations, to represent states of severe uncertainty. They have been successfully applied to a wide range of fields where risk under severe uncertainty is a concern. Following Walley's work from the 90's, the canonical interpretation of these bounds in the last 30 years has been through betting. However, in high-risk situations where the assessor themselves is at risk, various authors have argued that the betting interpretation of probability, and therefore also of lower previsions, cannot be applied. For this reason, in 2006, Lindley suggested an alternative interpretation of probability, based on urns, which mathematically leads to a theory of rational valued probabilities for finite possibility spaces. We revisit this interpretation in the context of non-linear expectations and lower previsions in particular.

In doing so, we (i) show how Lindley's interpretation can be generalized to infinite possibility spaces and arbitrary real-valued probabilities, effectively resulting in a finitely additive probability theory, and (ii) we provide an alternative interpretation of lower previsions, which leads to new expressions for consistency (called avoiding sure loss) and inference (called natural extension). We show that these expressions are mathematically equal to their canonical betting-based equivalents. We do this through a non-standard treatment of Lindley's urn interpretation based on internal set theory. This can be seen as an instance of Nelson's radically elementary probability approach.

As a side result, we establish a strong connection between Nelson's radically elementary non-standard probabilities and finitely additive probability measures.

References

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Alexander VERETENNIKOV

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On 2-recurrence for an SDE with switching

Abstract: Consider an SDE with switching in $\mathbb{R}^d$ for the component X,

$$dX_t = b(X_t, Z_t)\, dt + dW_t, \quad t \geq 0, \quad X_0 = x,\ Z_0 = z, \tag{1}$$

while Z_t is a continuous-time Markov process on the state space $S = \{0, 1\}$ with constant positive intensities of respective transitions $\lambda_{01} =: \lambda_0$, & $\lambda_{10} =: \lambda_1$; the trajecotry of Z is independent of the d-dimensional Wiener process W. Denote $b(x, 0) = b_-(x), \quad b(x, 1) = b_+(x)$, The boundedness condition for the Borel function b suffices for the process (X_t, Z_t) to be well-defined as a strong solution of the equation. Assume that there exist $M, r_\pm, R_\pm$ such that

$$xb_-(x) \leq -r_-, \quad xb_+(x) \leq +r_+, \quad \forall\, |x| \geq M, \tag{2}$$

and

$$\lambda_1(4r_- - 6d) > \lambda_0(4r_+ + 6d), \tag{3}$$

and

$$xb_-(x) \geq -R_-, \quad xb_+(x) \geq +R_+, \quad \forall\, |x| \geq M. \tag{4}$$

with $R_+ \leq r_+$, $R_- \geq r_- > 0$.

Theorem 1 *Let b be bounded and conditions (2)–(4) be satisfied. Then the process (X, Z) is 2-recurrent; for any $\delta > 0$ there exists $C > 0$ such that for any M_1 large enough and all $x \in \mathbb{R}^d$ and for $z = 0, 1$*

$$\mathbb{E}_{x,z}\tau_{M_1}^2 \leq C(x^{4+\delta} + 1),$$

where $\tau_{M_1} := \inf(t \geq 0 : |X_t| \leq M_1)$.

This implies that this bound holds true for *any* $M_1 > 0$ with a new constant C; also, similar bounds may be obtained for higher moments of the stopping time τ_{M_1} as well under further restrictions of recurrence conditions on $r_\pm$.

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A General Framework for Solving Singular SPDEs

Abstract: We propose a general framework of *proper regularization* to solve singular nonlinear SPDEs. Besides singularities in drift, this framework also allows singular noise coefficients including high order pseudo-differential operators. As applications, the (local and global) well-posedness is presented for a broad class of fluid dynamics equations with singular noise, such as the stochastic magnetohydrodynamics (including Navier-Stokes/Euler) equation, stochastic Korteweg-De Vries equation, stochastic (modified) Camassa-Holm type equations, stochastic aggregation-diffusion type equations and stochastic surface quasi-geostrophic equation. Thus, some recent results derived in the literature are extended in a unified way to the singular noise case. This is a joint work with Hao Tang.

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The Harnack inequality of stochastic differential equations.

Abstract: In this talk, I will present the dimension-free Harnack inequality of stochastic differential equations and stochastic differential delay equations for regular coefficients. Moreover, I will also discuss the dimension-free Harnack inequalities for stochastic equations with Hölder continuous diffusion coefficient and singular drift term without regularity assumption.

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Stochastic partial differential equations with local monotone coefficients

Abstract: Considered in a Gelfand triple, the well posedness of stochastic partial differential equations with monotone or particular type of local monotone coefficients is now well understood. In this talk, we will report recent progresses on the well-posedness of stochastic partial differential equations which have fully local monotone coefficients. The results apply to many interesting models/examples. This is a joint work with Michael Rockner and Shijie Shang.